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Standard Guide for Industrial Woven Wire Filter Cloth[1]

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INTRODUCTION

Industrial metal filter cloth is a special type of woven wire cloth that can be produced in many specifications, often proprietary in nature. Sometimes referred to as Dutch weave or Hollander weave, filter cloth can be woven in a variety of metals and is woven with a greater number of wires in one direction than the other, and utilizing two different wire diameters. This guide covers woven wire filter cloth for industrial use, which is commonly rated by its micron retention capability. Its purpose is to introduce standard terms and definitions, to observe common technical considerations that a user should be aware of, and to present a mathematical model that can be used to predict the micron retention of a filter cloth specification. It should be noted this guide excludes standard industrial woven wire cloth and sieve cloth from its scope, since these are covered under Specifications E2016 and E11, respectively, as well as excludes plastic and synthetic filter cloth.

1. Scope

1.1 This guide covers the special grade of industrial woven wire cloth, referred to as filter cloth, for general filtration including the separation of solids from fluids (liquids or gases), based on a desired particle size retention. Filter cloth can be made of any primary metal or metal alloy wire that is suitable for weaving.

1.2 The values stated in inch-pound units are to be regarded as standard. The values given in parentheses are mathematical conversions to SI units that are provided for information only and are not considered standard.

1.3 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

E11 Specification for Woven Wire Test Sieve Cloth and Test Sieves
E1638 Terminology Relating to Sieves, Sieving Methods, and Screening Media
E2016 Specification for Industrial Woven Wire Cloth
F316 Test Methods for Pore Size Characteristics of Membrane Filters by Bubble Point and Mean Flow Pore Test

2.2 *SAE Standards:*[3]

ARP901 Bubble-Point Test Method

3. Terminology

3.1 *Definitions:*

3.1.1 For additional terminology, refer to Terminology E1638.

3.1.2 *bubble point test, n*—capillary flow bubble point methods are based on the fact that the pressure required to force an air bubble through filter cloth wetted under a test liquid of known surface tension is inversely proportional to the pore size.

3.1.2.1 *Discussion*—The pressure observed at the first bubble location is considered the absolute micron retention rating (see Test Method F316).

3.1.3 *cloth thickness, n*—overall thickness of the filter cloth, nominally estimated by adding the warp wire diameter plus two times the shute wire diameter.

3.1.4 *crimp, n*—corrugation in the warp and shute wires.

3.1.4.1 *Discussion*—The crimp in the wires is formed during the weaving process, and the tension existing between the warp and shute wires fundamentally determines the respective

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[1] This guide is under the jurisdiction of ASTM Committee E29 on Particle and Spray Characterization and is the direct responsibility of Subcommittee E29.01 on Sieves, Sieving Methods, and Screening Media.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

[3] Available from SAE International (SAE), 400 Commonwealth Dr., Warrendale, PA 15096-0001, http://www.sae.org.

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amount or depth of crimp, which in part establishes the firmness of the filter cloth. With the exception of reverse filter cloth, the warp wire is tensioned such that it only crimps minimally if at all, and the shute wire crimps predominately around the warp wire.

3.1.5 *filter cake (surface cake), n*—material that is retained on the filter cloth during processing.

3.1.5.1 *Discussion*—The filter cake forms and builds up as particulate is retained, until the increased flow resistance of the filter cake requires it be removed from the filter cloth, typically by backflushing. The deposition of material forming the filter cake can aid in filtration by providing depth filtration, which results in a lower micron retention.

3.1.6 *glass bead test, n*—method for determining the filtration rating of filter cloth using a set of presorted precisely sized spherical glass beads, passing them through the filter cloth, and examining the beads passed or captured.

3.1.6.1 *Discussion*—The largest bead passed is considered the absolute micron retention rating.

3.1.7 *mesh, n*—number of wires or openings per linear inch or 25.4 mm counted from the center of any wire to a point exactly 1 in. or 25.4 mm distant, including the fractional distance between either thereof.

3.1.8 *micron, n*—common filtration reference to a particle size, properly defined as a micrometre.

3.1.9 *micron retention, n*—separation particle size of the filter cloth expressed as a diameter in micrometres.

3.1.10 *micron retention, absolute, n*—diameter of the largest spherical particle that will pass through the filter cloth under laboratory conditions representing the maximum pore size.

3.1.11 *micron retention, nominal, n*—subject to user definition, an indication of the average pore size of the filter cloth.

3.1.11.1 *Discussion*—The nominal rating may refer to: (*1*) the glass bead or particle size the filter cloth will retain 90 % of by weight; (*2*) the bubble point pore size when the tenth bubble location appears; or (*3*) the degree of filtration achieved under specific process conditions such as operating pressure, concentration of contaminant, and the buildup of filter cake, such that 94 % to 98 % of all particles of the nominal value will be retained after a given working period.

3.1.12 *percent open area, n*—because of the irregular triangular-shaped opening formed at an angle to the plane of the filter cloth surface, the percent open area is generally not a specified parameter.

3.1.13 *shute wires, n*—wires running the short way of, or across the cloth, as woven (also referred to as the shoot, fill, or weft wires).

3.1.14 *types of weaves, n*—

3.1.14.1 *double warp, adj*—filter cloth (either plain or twill) in which two warp wires are used instead of one for each warp pitch thus reducing the micron retention of a similar regular single-warp wire specification (see Fig. 1).

3.1.14.2 *plain, adj*—filter cloth in which the shute wires pass over one and under one warp wire (see Fig. 2).

3.1.14.3 *reverse weave, adj*—filter cloth in which the warp and shute wires are woven in a reverse configuration; not covered within this guide (see Fig. 3).

3.1.14.4 *twill, adj*—filter cloth in which the shute wires pass over two and under two wires (see Fig. 4).

3.1.15 *warp wires, n*—the wires running the long way of the cloth as woven.

3.1.16 *weight per unit area, n*—weight per square foot for filter cloth can be approximated (without consideration for the significant crimp of the shute wire) by the following equation:

$$Wt/ft^2 = \left[12 M_w \left(12\pi \left(D_w^{\,2}/4\right) \rho\right)\right] + \left[12 M_s \left(12\pi \left(D_s^{\,2}/4\right) \rho\right)\right] \quad (1)$$

where:

Wt/ft^2 = weight (lb) per square foot,
M_w = mesh warp,
M_s = mesh shute,
D_w = diameter warp wire,
D_s = diameter shute wire,
ρ = density of material ($lb/in.^3$) (0.2836 for stainless steel 304),
π = constant 3.1416.

3.1.16.1 *Discussion*—The theoretical mass per unit area can be similarly calculated with SI units or an approximate multiplier factor of 4.8824 can be used to obtain kilograms per square metre.

3.1.17 *wire diameter, n*—wire diameter shall be expressed in decimal parts of an inch or the metric equivalent.

4. Significance and Use

4.1 Industrial filter cloth is a specialized product that can be manufactured in many specifications. The purpose of this guide is to (*1*) introduce standard terms and definitions associated with wire filter cloth, (*2*) observe common technical considerations that a user should be aware of, and (*3*) present a mathematical model that can be used to predict the micron

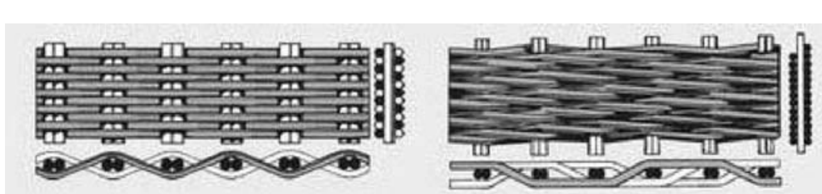

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FIG. 1 Double Warp Plain and Double Warp Twill Weave

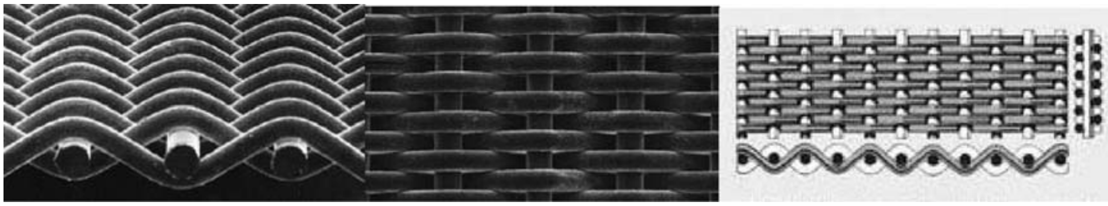

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FIG. 2 Plain Weave

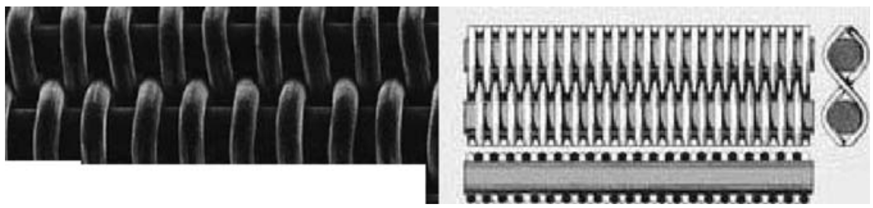

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FIG. 3 Reverse Plain Weave

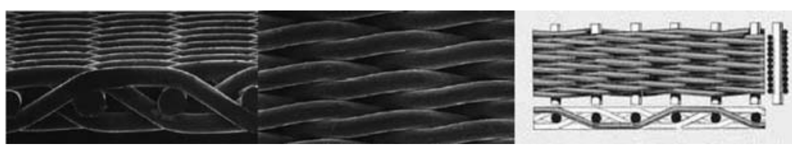

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FIG. 4 Twill Weave

retention of a filter cloth specification. As often numerous specifications may be developed to result in a common micron retention by varying the weave type, mesh count, and wire diameters, it is recommended that the user consult with their filter cloth supplier regarding specific filter cloth specifications of interest and include in their discussions durability, pressure drop, and cleaning capability requirements. The purpose of this guide is not to suggest a limited selection of specifications.

4.2 The micron retention of a filter cloth specification can be mathematically modeled as well as determined by the use of a glass bead test or the bubble-point test method or both depending on the degree of fineness. Typical standard bubble-point test methods (porometry) include Test Methods F316 and SAE ARP901.

5. Filter Cloth Specifications

5.1 Filter cloth is woven in a variation of sometimes proprietary parameters based on often common nominal mesh count specifications. This is due to minor variations in mesh count and wire diameters used to affect micron retention, porosity, and other factors related to specific operating conditions, as well as possibly for manufacturing convenience. Therefore, it is not appropriate to provide a comprehensive table of common filter specifications stating construction requirements and resulting parameters. Instead, a mathematical model is presented that can be used to predict the micron retention or separation particle size of any filter cloth specification a user and producer wish to develop.

5.2 This mathematical model is presented by Reiner Tittel and Rolf Berndt[4] with further conclusions by Denis Blackmore (see Appendix X1). The model assumes rigid, spherical particles that pass through various planes or cross sections of the filter cloth created by shute wires stretched around warp wires and positioned geometrically adjacent to one another. The separation particle size is determined for the applicable geometric plane based on the weave type and specification ratios.

5.3 While five geometric planes of the filter cloth are considered (three of interest as the outer two are symmetrical), Plane 3, designated the geometric middle plane of the filter cloth, is the primary plane of interest. Accordingly, the separation particle size (dTr_3) is determined for plain weave with warp wire to shute wire diameter ratios within the range 1.00 to 1.50 (see Annex A1). For twill weave with warp pitch to warp wire diameter ratios greater than 3.22, Plane 2 is considered and the separation particle size (dTr_2) is determined. For the calculation of dTr_2, Blackmore concludes that for the equated Tittel and Berndt equations, the coordinate origin ratio (t/t_1) and the geometric dimension (x) can both be expressed as a function of the warp-to-shute-wire diameters (b) (see Annex A2). The model is not applicable for reverse weave filter cloth.

5.4 A selection of typical woven wire filter cloth specifications are presented with their particle size retentions as determined by the Tittel and Berndt model in conjunction with

[4] Tittel, R. and Berndt, R., “Zur bestimmung der trennteilchengr ße von filtergeweben,” *Faserforschung und Textiltechnik* , Vol 24, 1973, pp. 505–510.

the Blackmore conclusions (see Appendix X2). These specifications are only for example, as countless others may be considered for weaving. Check with a weaver to determine the feasibility of others. Note that the physical properties of the wire to be woven may have an impact on overall filter cloth quality.

6. Technical Considerations

6.1 *Wire Tolerances*—The diameter tolerance for wire before weaving commonly should be in accordance with industrial standards. It is recognized that mechanical deformation of particularly the shute wire occurs during weaving. Therefore, the diameter measured after weaving can only be used as a guide to the original nominal diameter.

6.2 *Filter Cloth Tolerances:*

6.2.1 Industrial filter cloth can be woven from a great variety of metals and alloys. Metals other than 300 series stainless steel may not be possible depending on the particular specification and should be discussed with the supplier.

6.2.2 Tolerances on parameters such as mesh count and micron retention should be discussed with the supplier.

6.2.3 The shute wires should be stretched around the warp wires and positioned adjacent to one another. Irregular gaps between the shute wires may indicate irregular retention capability.

6.3 *Filter Cloth Blemishes*—Filter cloth may exhibit some blemishes or defects that are inherent to the weaving process. The permissible number of major blemishes or defects should be discussed with the supplier. Any irregular opening in an area of filter cloth, as a result of any various cause, shall be considered a defect if the agreed to micron retention is exceeded.

6.4 *Delivery Requirements:*

6.4.1 Except when specifically agreed to otherwise, the total quantity of filter cloth furnished should be within ±10 % of the quantity ordered. The invoice should be based on the actual quantity furnished.

6.4.2 A standard roll is 100 linear feet (30.5 m) ± 10 linear feet (3 m), but each specification should be discussed with the supplier.

6.4.3 The nominal width of the roll should be specified, as well as the permissible tolerance if applicable, and whether the roll may be delivered with or without selvage edges.

6.4.4 The percentage of yield of the filter cloth shall be agreed on with the customer and will vary according to the specification and size of the product.

6.4.5 The flatness of woven filter cloth with regard to both curl and waviness should be discussed with the supplier.

6.4.6 Some filter cloth specifications may exhibit frayed edges.

6.4.7 Firmness is a subjective term referring to the planar rigidity of filter cloth established by the tensile strength of the material, the relationship of the mesh to wire diameters, the type of weave, the amount of crimp in the wires, and the tension on the warp wires during the weaving. The absence of firmness in woven wire filter cloth is termed sleaziness. Woven wire filter cloth should normally exhibit satisfactory firmness; that may be discussed with the supplier.

6.4.8 Woven filter cloth may be covered with a film of oil or other lubricant as a result of the manufacturing process. The wire may show traces of products used in or markings caused by the drawing process.

6.4.9 The tolerances that can be held on cut-to-size pieces of filter cloth can be dependent on the piece size, the mesh, wire diameters, type of weave, and firmness of the weave. These factors should be considered in the discussion of tolerances with the supplier.

7. Procedure

7.1 Filter cloth is best inspected using a backlight to observe irregular and defective openings.

7.2 The mesh count of filter cloth may be checked using a counting glass compatible with the degree of fineness. All test apparatus should be calibrated against standards traceable to the National Institute of Standards and Technology.

8. Packaging and Labeling

8.1 *Packaging*—Depending on the specification, woven filter cloth may be rolled on a wooden or cardboard roll or more durable specifications without a center roll, but in any case, the method of packaging should take into account the likelihood of being damaged. Any special packaging should be specified and agreed to with the supplier.

8.2 *Labeling:*

8.2.1 Filter cloth should be labeled with the following information:

8.2.1.1 The name of the manufacturer;

8.2.1.2 The material of the wire;

8.2.1.3 The mesh designation of the specification;

8.2.1.4 The type of weave; and

8.2.1.5 The quantity, that is, length and width, or the size and number of pieces.

8.2.2 Other labeling requirements may be subject to agreement between the customer and the supplier.

9. Certification

9.1 At the time of ordering, customers may request a test certificate containing the following information or parts thereof:

9.1.1 *Chemical Analysis of the Weaving Wires*—For the chemical analysis of the material, the wire manufacturer's batch, heat, or melt number analysis is applicable.

9.1.2 Mesh count or additional tests as agreed between the customer and the supplier.

10. Keywords

10.1 Dutch weave; filter cloth; micron retention; wire cloth

ANNEXES

(Mandatory Information)

A1. CALCULATION OF dTr_3 FOR SEPARATION PARTICLE SIZE IN ACCORDANCE WITH TITTEL AND BERNDT (1973)[4]

A1.1 For 24 × 110 mesh plain:

A1.1.1 Pitch warp wires (t_1):

$$t_1 = 1/24 = 0.0417\text{ in.} = 1.0583\text{ mm}$$

A1.1.2 Warp wire diameter (d_k):

$$d_k = 0.015\text{ in.} = 0.381\text{ mm}$$

A1.1.3 Shute wire diameter (d_s):

$$d_s = 0.010\text{ in.} = 0.254\text{ mm}$$

A1.1.4 Ratio of warp to shute wire diameters (b):

$$b = d_k/d_s = 1.50$$

A1.2 For the "Plane 3" pore triangle:

A1.2.1 Base of triangle (g):

$$g = d_s*\left(\frac{1+(1+b)^2*0.66*(1-b*d_s/t_1)}{[1+(1+b)^2*0.436]^{0.5}} - 1\right)$$

$$g = 0.2250$$

A1.2.2 Height of triangle (h_t):

$$h_t = t_1*0.5*\left(1+\frac{1-[1+(1+b)^2*0.436]^{0.5}}{(1+b)^2*0.66} - (b*d_s/t_1)\right)$$

$$h_t = 0.2194$$

A1.2.3 Particle size (dTr_3):

$$dTr_3 = g*\{[(g/(2*h_t))^2+1]^{0.5} - g/(2*h_t)\}$$

$$dTr_3 = 0.137\text{ mm} = 137\ \mu\text{m}$$

A2. CALCULATION OF dTr_2 FOR SEPARATION PARTICLE SIZE IN ACCORDANCE WITH TITTEL AND BERNDT (1973)[4] WITH BLACKMORE (2009) (Appendix X1)

A2.1 For 20 × 250 mesh twill:

A2.1.1 Pitch warp wires (t_1):

$$t_1 = 1/20 = 0.050\text{ in.} = 1.270\text{ mm}$$

A2.1.2 Warp wire diameter (d_k):

$$d_k = 0.010\text{ in.} = 0.254\text{ mm}$$

A2.1.3 Shute wire diameter (d_s):

$$d_s = 0.0085\text{ in.} = 0.216\text{ mm}$$

A2.1.4 Ratio of warp to shute wire diameters (b):

$$b = d_k/d_s = 1.1765$$

A2.1.5 Ratio of warp pitch to warp wire diameter:

$$t_1/d_k = 5.00$$

A2.2 For the "Plane 2" pore triangle:

A2.2.1 Coordinate origin ratio (t/t_1):

$$t/t_1 = [(1+b)^2+3]/[2*((1+b)^2+1)]$$

$$t/t_1 = 0.6743$$

A2.2.2 Geometric dimension (x):

$$x = d_s*\left(\frac{[(1+b)^6+7*(1+b)^4+7*(1+b)^2+1]^{0.5}}{2*((1+b)^2+1)} - 1\right)$$

$$x = 0.1087$$

A2.2.3 Particle size (d_0):

$$d_0 = d_s*(\{(x/d_s+1)^2/[(x/d_s+1)^2 - 0.25]^{0.5}\} - 1)$$

$$d_0 = 0.1283$$

A2.2.4 Correction factor (Z):

$$Z = t_1/[t_1^2 - d_s^2*(1+b)^2]^{0.5}$$

$$Z = 1.0764$$

A2.2.5 Particle size (dTr_2):

$$dTr_2 = d_0 - [0.4*d_s*(Z-1)]$$

$$dTr_2 = 0.122\text{ mm} = 122\ \mu\text{m}$$

APPENDIXES

(Nonmandatory Information)

X1. DENNIS BLACKMORE NOTES ON TITTEL AND BERNDT (1973)[4]

Notes on the Tittel-Berndt Filter Fabric Model

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December 21, 2009

X1.1 Introduction

X1.1.1 Our purpose is to clarify some points in the model developed in Tittel and Berndt (1973). In particular, we shall try to clarify the derivations of Equations (2) and (3) of Tittel and Berndt, which we believe are presented in the wrong order and therefore mislabeled (although ultimately the order is irrelevant). This can be accomplished by imbedding the various elements of the fabric—namely the basic fabric planes and various weft wires (in Figure 3 and Figure 5 of Tittel and Berndt)—in a three-dimensional euclidean coordinate system.

X1.2 Euclidean Coordinate Representation of Fabric Configuration

X1.2.1 We start by introducing cartesian coordinates (X, Y, Z) in 3-space $\mathbb{R}^3$ so that the pore plane lies in the XY-plane. More precisely, we position the origin in our coordinate system at a point in the pore plane $\mathcal{P}$ midway between the warp wires K_1 and K_2 (having diameters d_k) so that the parallel axes of symmetry of these wires are parallel to the X-axis (the position of the origin corresponds to the dark dot in the upper portion of Figure 5 of Tittel and Berndt (1973)). Then the wires are represented as the solid cylinders:

$$K_1 := \left\{ \mathrm{X} = (X, Y, Z) : \left(Y - \frac{t_1}{2} \right)^2 + Z^2 \le \left(\frac{d_k}{2} \right)^2 \right\} \qquad (X1.1)$$

$$K_2 := \left\{ \mathrm{X} = (X, Y, Z) : \left(Y + \frac{t_1}{2} \right)^2 + Z^2 \le \left(\frac{d_k}{2} \right)^2 \right\}$$

and the pore plane is characterized by:

$$\mathcal{P} := \{ \mathrm{X} = (X, Y, 0) : (X, Y) \in \mathbb{R}^2 \} \qquad (X1.2)$$

X1.2.2 Next we have to find the representations of the weft wires in Figure 3 and Figure 5 of Tittel and Berndt (1973). We shall denote these cylindrical wires as w_1, w_2, and w_3 (each having a common diameter of d_s), and describe them with reference to Figure 3 and Figure 5. As the wires are (ideally) cylindrical tubes, they can be specified by their centerlines. We see from Figure 5 that the centerline of w_1—at least between its first intersection points with K_1 and K_2—has the (parametric) form:

$$\ell_1 : X(\alpha) = \left(\frac{d_s}{2}(1+\alpha), \frac{(1-2\alpha)t_1 - D}{2}, \frac{D}{2} \right) \qquad (X1.3)$$

where:

$$D := d_s + d_k$$

and the parameter α (not to be confused with the angle in Figure 4 of Tittel and Berndt (1973)) ranges over all real numbers, and the wire is tangent to K_1 when $\alpha = \alpha_1 := -(2t_1)^{-1} D$ and tangent to K_2 when $\alpha = \alpha_2 := 1 + \alpha_1$. Similarly, we find from Figure 3 that the centerlines ℓ_2 and ℓ_3 of w_2 and w_3, respectively (between their first intersection points with K_1 and K_2), have the parametric forms:

$$\ell_2 : X(\beta) = \left(-\frac{d_s}{2}(1+\beta), \frac{(t_1 - D)}{2}(1-2\beta), D\sqrt{\frac{t_1 - D}{t_1 + D}}(1-2\beta) \right) \qquad (X1.4)$$

$$\ell_3 : X(\gamma) = \left(\frac{d_s}{2}(1-\gamma), \frac{(t_1 + D)}{2}(1-2\gamma), -D\sqrt{\frac{t_1 + D}{t_1 - D}}(1-2\gamma) \right)$$

X1.2.3 We note here that these equations were obtained using 3D analytic geometry, with a large assist from some plane geometry and trigonometry of the configuration in a number of planes perpendicular to the X-axis.

X1.3 Calculation of Key Distances

X1.3.1 It now rather straightforward to calculate the key quantity:

$$\left(\frac{x}{d_s} \right)$$

which is defined to be the common value of the distance between a point on ℓ_1 and ℓ_2, divided by d_s, denoted as:

$$\left(\frac{x}{d_s} \right)_{1/2}$$

and the distance between a point on ℓ_1 and ℓ_3, divided by d_s, denoted as:

$$\left(\frac{x}{d_s} \right)_{1/3}$$

when the two are equal.

X1.3.2 We need only consider these distances for points in the configuration on the wires w_1, w_2, and w_3 lying between the wires K_1 and K_2, so the equations for the centerlines are given by the equations Eq X1.3 and Eq X1.4. Now the distance between a point X_1 on ℓ_1 and the line ℓ_2 (ℓ_3) is the distance between X_1 and the point X_2 on ℓ_2 (X_3 on ℓ_3) such that the line through X_1 and X_2 (X_3) is orthogonal or perpendicular to the line ℓ_2 (ℓ_3). Using the representations above, we see that if we fix the parameter α and with it a point $X(\alpha)$ on ℓ_1 the orthogonality condition requires that we choose $X(\beta) \in \ell_2$ and $X(\gamma) \in \ell_3$ such that:

$$X(\beta) - X(\alpha) = \left(-\frac{d_s}{2}\ (1+\alpha+\beta)\ ,\ \ \alpha\ t_1\ -\ \beta\ (t_1 - D)\ ,\ \frac{D}{2}\ \left[\ \sqrt{\frac{t_1 - D}{t_1+D}}\,(1 - 2\beta) - 1\right]\ \right) \perp \ell_2 \tag{X1.5}$$

and:

$$X(\gamma) - X(\alpha) = \left(-\ \frac{d_s}{2}\ \ (\alpha+\gamma)\ ,\ \ \alpha\ t_1\ -\ \gamma\ (t_1 - D)\ +\ D\ ,\ \frac{D}{2}\ \left[\ \sqrt{\frac{t_1+D}{t_1 - D}}\,(1 - 2\gamma) - 1\right]\ \right) \perp \ell_3 \tag{X1.6}$$

where the symbol $\perp$ as usual denotes the relation of being orthogonal. But (Eq X1.5) and (Eq X1.6) are respectively equivalent to:

$$(X(\beta) - X(\alpha))\cdot\frac{dX(\beta)}{d\beta} = 0$$

and:

$$(X(\gamma) - X(\alpha))\cdot\frac{dX(\gamma)}{d\gamma} = 0$$

where $\cdot$ is the usual dot or inner or scalar product in $\mathbb{R}^3$, so it follows that:

$$\begin{aligned} 0 &= (X(\beta) - X(\alpha))\cdot\frac{dX(\beta)}{d\beta} \\ &= (X(\beta) - X(\alpha))\cdot\left(-\frac{d_s}{2}, -(t_1 - D), -2D\sqrt{\frac{t_1 - D}{t_1+D}}\right) \\ &= \left(\frac{d_s}{2}\right)^2 (1+\alpha+\beta) - (t_1 - D)\,[\alpha t_1 - \beta(t_1 - D)] \\ &\quad -D^2\sqrt{\frac{t_1 - D}{t_1+D}}\left[\ \sqrt{\frac{t_1 - D}{t_1+D}}\,(1 - 2\beta) - 1\right] \end{aligned} \tag{X1.7}$$

and:

$$0 = (X(\gamma) - X\,(\alpha))\cdot\frac{dX(\gamma)}{d\gamma} \tag{X1.8}$$

$$\begin{aligned} &= (X(\gamma) - X(\alpha))\cdot\left(-\frac{d_s}{2}, -(t_1+D), 2D\sqrt{\frac{t_1+D}{t_1 - D}}\right) \\ &= \left(\frac{d_s}{2}\right)^2 (\alpha+\gamma) - (t_1+D)\,[\alpha t_1 - \gamma(t_1 - D)+D] \\ &\quad +D^2\sqrt{\frac{t_1+D}{t_1 - D}}\left[\ \sqrt{\frac{t_1+D}{t_1 - D}}\,(1 - 2\gamma) - 1\right] \end{aligned}$$

X1.3.3 Next for the fixed α, we solve for β and γ from (Eq X1.7) and (Eq X1.8), respectively, to obtain:

$$\beta = \beta(\alpha) = \frac{-(t_1+D)\ \{[(d_s/2)^2 - t_1\ (t_1 - D)]\ \alpha+[(d_s/2)^2+D^2\ R\ (R - 1)]\}}{[(d_s/2)^2\ (t_1+D)+(t_1 - D)\ (t_1^2+D^2)]} \tag{X1.9}$$

where:

$$R := \sqrt{\frac{t_1 - D}{t_1+D}}$$

and:

$$\gamma = \gamma(\alpha) = \frac{-(t_1 - D)\ \{[(d_s/2)^2 - t_1\ (t_1+D)]\ \alpha - D\ [(t_1+D) - DR^{-1}\ (R^{-1} - 1)]\}}{[(d_s/2)^2\ (t_1 - D)+(t_1+D)\ (t_1^2 - 2t_1D+D^2)]} \tag{X1.10}$$

X1.3.4 Accordingly we find that the nearest points to X(α) on the lines ℓ_2 and ℓ_3 are respectively:

$$X(\beta(\alpha)) = \left(-\frac{d_s}{2}\,(1+\beta(\alpha)), \frac{(t_1 - D)}{2}\,(1 - 2\beta(\alpha)), D\sqrt{\frac{t_1 - D}{t_1+D}}\,(1 - 2\beta(\alpha))\right)$$

and:

$$X(\gamma(\alpha)) = \left(\frac{d_s}{2}\,(1 - \gamma(\alpha)), \frac{(t_1+D)}{2}\,(1 - 2\gamma(\alpha)), -D\sqrt{\frac{t_1+D}{t_1 - D}}\,(1 - 2\gamma(\alpha))\right)$$

where β(α) and γ(α) are given by (Eq X1.9) and (Eq X1.10), respectively.

X1.3.5 Whence we calculate the (euclidean) distances between the centerlines as:

$$\begin{aligned} x_{1/2} &:= \|X(\beta(\alpha)) - X(\alpha)\| \\ &:= [(X(\beta(\alpha)) - X(\alpha))^2+(Y(\beta(\alpha)) - Y(\alpha))^2+(Z(\beta(\alpha)) - Z(\alpha))^2]^{1/2} \\ &= \left\{\left[\left(\frac{d_s}{2}\right)(1+\alpha+\beta(\alpha))\right]^2+[\alpha t_1 - \beta(\alpha)(t_1 - D)]^2+\left[\frac{D}{2}\,[R\,(1 - 2\beta(\alpha)) - 1]\right]^2\right\}^{1/2} \end{aligned} \tag{X1.11}$$

and:

$$\begin{aligned} x_{1/3} &:= \|X(\gamma(\alpha)) - X(\alpha)\| \\ &:= [(X(\gamma(\alpha)) - X(\alpha))^2+(Y(\gamma(\alpha)) - Y(\alpha))^2+(Z(\gamma(\alpha)) - Z(\alpha))^2]^{1/2} \\ &= \left\{\left[\left(\frac{d_s}{2}\right)(\alpha+\gamma(\alpha))\right]^2+[\alpha t_1 - \gamma(\alpha)(t_1 - D)+D]^2+\left[\frac{D}{2}\,[R^{-1}\,(1 - 2\gamma(a)) - 1]\right]^2\right\}^{1/2} \end{aligned} \tag{X1.12}$$

X1.4 Final Calculations

X1.4.1 To complete our analysis it remains to equate (Eq X1.11) and (Eq X1.12), or better yet to equate their squares. This will enable us to determine the parameter value of α so that the point X(α) on ℓ_1 is such that it is equidistant from both ℓ_2 and ℓ_3. In Tittel and Berndt (1973) this common value is denoted as *x*, so that:

$$\left(\frac{x}{d_s}\right) = \left(\frac{x}{d_s}\right)_{1/2} = \left(\frac{x}{d_s}\right)_{1/3} \tag{X1.13}$$

and this common value may then be substituted in Equation (5) of Tittel and Berndt in order to find the ratio of the theoretical particle size (diameter) to the diameter of the weft wires:

$$\frac{d_0}{d_s}$$

X1.4.2 Before equating the squares of (Eq X1.11) and (Eq X1.12), it is convenient to rewrite (Eq X1.9) and (Eq X1.10) in the form:

$$\beta = \beta(\alpha) = A_2\alpha+B_2 \tag{X1.14}$$

and:

$$\gamma = \gamma(\alpha) = A_3\alpha + B_3 \tag{X1.15}$$

where the definitions of the constant coefficients A_2, A_3, B_2, and B_3 follow directly from (Eq X1.9) and (Eq X1.10); namely:

$$A_2 := \frac{-(t_1+D)\left[(d_s/2)^2 - t_1(t_1+D)\right]}{\left[(d_s/2)^2(t_1-D)+(t_1+D)(t_1^2-2t_1D+D^2)\right]} \tag{X1.16}$$

$$A_3 := \frac{-(t_1-D)\left[(d_s/2)^2 - t_1(t_1+D)\right]}{\left[(d_s/2)^2(t_1-D)+(t_1+D)(t_1^2-2t_1D+D^2)\right]}$$

$$B_2 := \frac{-(t_1+D)\left[(d_s/2)^2 + D^2R(R-1)\right]}{\left[(d_s/2)^2(t_1+D)+(t_1-D)(t_1^2+D^2)\right]}$$

$$B_3 := \frac{D(t_1-D)\left[(t_1+D) - DR^{-1}(R^{-1}-1)\right]}{\left[(d_s/2)^2(t_1-D)+(t_1+D)(t_1^2-2t_1+D^2)\right]}$$

X1.4.3 Whence, equating the spares of (Eq X1.11) and (Eq X1.12) yields:

$$\begin{aligned} d_s^2&\left[(A_2+1)\ \alpha + (B_2+1)\right]^2 + 4\left[((D-t_1)A_2+t_1)\alpha\right. \\ &\left.+(D-t_1)B_2\right]^2 + D^2\left[-2RA_2\alpha + (R(1-2B_2)-1)\right]^2 \\ =\ & d_s^2\left[(A_3+1)\alpha+B_3\right]^2 + 4\left[((D-t_1)A_3+t_1)\alpha\right. \\ &\left.+((D-t_1)B_3+D)\right]^2 + D^2\left[-2R^{-1}A_3\alpha\right. \\ &\left.+(R^{-1}\ (1-2B_3)\ -\ 1\right]^2 \end{aligned}$$

which is the equivalent to the quadratic equation (in α):

$$\begin{aligned} 0 = &\{d_s^2\left[(A_2+1)^2-(A_3+1)^2\right] + 4\left[((D-t_1)A_2+t_1)^2 - ((D\right. \\ &\left.-t_1)A_3+t_1)^2\right] + 4D^2(R^2A_2^2 - R^{-2}A_2^3)\}\alpha^2 + 2\{d_s^2\left[(A_2+1)(B_2\right. \\ &\left.+1) - 2\{A_3+1\}B_3\right] + 4\left[((D-t_1)A_2+t_1)(D-t_1)B_2 - ((D\right. \\ &\left.-t_1)A_3+t_1)((D-t_1)B_3+D)\right] - 4D^2\left[RA_2(R(1-2B_2)-1)\right. \\ &\left.-R^{-1}A_3(R^{-1}(1-2B_2)-1)\right]\}\alpha + \{d_s^2\left[(B_2+1)^2-B_3^2\right] + 4\left[(D\right. \\ &\left.-t_1)^2B_2^2 - ((D-t_1)B_3+D)^2 + \left[(R(1-2B_2)-1)^2 - (R^{-1}(1\right.\right. \\ &\left.\left.-2B_3)-1)^2\right]\right]\} \end{aligned} \tag{X1.17}$$

which we write in the form:

$$C_2\alpha^2 + C_1\alpha + C_0 = 0 \tag{X1.18}$$

where the coefficients are defined as in (Eq X1.17). If $C_2 = 0$, the equation has a unique positive solution that we denote as α_*. On the other hand, when $C_2 \neq 0$, we define α_* to be the smallest positive number of the form the follows from the quadratic root equation:

$$\alpha_* = \frac{-C_1 \pm \sqrt{C_1^2 - 4C_0C_2}}{2C_0} \tag{X1.19}$$

X1.4.4 Finally, we obtain the desired equation:

$$\frac{x}{d_s} = \frac{1}{d_s}\left\| X(\beta(\alpha_*)) - X(\alpha_*)\right\| = \frac{1}{d_s}\left\| X(\gamma(\alpha_*)) - X(\alpha_*)\right\| \tag{X1.20}$$

with the norms defined as in (Eq X1.11) and (Eq X1.12), which can be used to obtain d_0/d_s from Equation 5 of Tittel and Berndt (1973).

X1.5 Concluding Remarks

X1.5.1 My Eq X1.20 is similar to, but I think not exactly the same as Equation (4) of Tittel and Berndt (1973). The difference—if any—would lie in the exact positioning of the wires as described in the paper. In fact, the Equations (2) and (3) of Tittel and Berndt do not seem to agree exactly with the positions of the wires indicated in the paper (which I used to obtain my equations). For example, Equation (2) says that the centerlines of w_1 and w_3 (or do they mean w_1 and w_2?) intersect when $t = 0$; but from the figures, this seems to be impossible for w_1 and w_3 as well as for w_1 and w_2.

X1.5.2 By the way, if their equations are correct, then x/d_s is defined by the property:

$$(1+b)^2\left(\frac{t}{t_1}\right)^2 + \left(1-\frac{t}{t_1}\right)^2 = (1+b)^2\left(1-\frac{t}{t_1}\right)^2 + \left(2-\frac{t}{t_1}\right)^2$$

which implies:

$$\frac{t}{t_1} = \frac{(1+b)^2+3}{2\left[(1+b)^2+1\right]}$$

from which we conclude:

$$\frac{x}{d_s} = \frac{\sqrt{(1+b)^6 + 7(1+b)^4 + 7(1+b)^2 + 1}}{2\left[(1+b)^2+1\right]} - 1$$

X2. EXAMPLE CALCULATIONS

X2.1 See Table X2.1 for example calculations.

TABLE X2.1 Separation Particle Size for Typical Filter Cloth

Separation Particle Size for Typical Filter Cloth					
Mesh	Wire Diameter		Weave	Particle Size	
	Warp	Shute		Plane 3	Plane 2
12 × 64	0.023	0.0165	plain	283	
24 × 110	0.015	0.010	plain	137	
30 × 150	0.009	0.007	plain	113	
40 × 200	0.007	0.0055	plain	85	
50 × 250	0.0055	0.0045	plain	68	
50(2) × 250	0.0045	0.0045	plain	50	
20 × 200	0.0135	0.011	twill		155
20 × 250	0.010	0.0085	twill		122
20(2) × 250	0.0085	0.0085	twill		106
30 × 250	0.011	0.0082	twill	112	
80 × 700	0.004	0.003	twill	42	
120 × 500	0.004	0.0028	twill	23	
150 × 800	0.0028	0.002	twill	20	
200 × 1400	0.0023	0.0016	twill	14	

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